

## FUZZY MODELING IN GAME AI

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**ABSTRACT.** In this survey, we outline the impact fuzzy set theory had on artificial intelligence in games. Therefore, we will review fuzzy set related achievements in research and industrial applications alike. We will specifically address such topics as fuzzy game theory, fuzzy data analysis for real games, and the development of game AI agents using fuzzy models.

**Keywords:** fuzzy game theory, fuzzy data analysis, fuzzy modeling, game AI, believable agents

**AMS Subject Classification:** 68T02

### 1. INTRODUCTION

Since the proposal of fuzzy sets by Zadeh [121], fuzzy set theory [13, 122] had impacted research and development in many areas. By him laying down the mathematical foundations, the move from Boolean to fuzzy logic had been triggered. Taking the inherent fuzziness of linguistic information into account has proven useful in many control applications and allowed for more human-like models of reasoning and decision-making.

Games provide human players with complex decision-making tasks, which often require reasoning under not clearly defined conditions. Throughout the years, they have proven a useful benchmark for artificial intelligence (AI). Trying to win against the human world champions of Chess and Go has shown to be an accelerator of innovation in AI research and spawned many interesting developments in and outside the field of game research. The development of AI agents for games requires the handling of several interesting aspects for which Zadeh has created a tool that complements the already existing models. Those include but are not limited to: using graded possibilities to model uncertainty, integrating graded truths in modeling the game's state, and overall equipping the AI agents with behavior that allows for gradually switching between different behaviors and modeling preferences etc.

In this paper we are going to review which areas of fuzzy set theory are successful in real applications (data analysis, control) and which ones might become so (fuzzy game theory). Therefore, we are going to summarize how the development of fuzzy set theory impacted areas such as but not limited to game theory, game analysis, game development, and game optimization. While doing so we will discuss key concepts applied in these fields, the underlying theory, and in some cases their industrial applications and achievements.

The following sections are each dedicated to separate fields of research or application. Section 2 starts with an overview of game theory and how the introduction of fuzzy set theory has changed the way in which games are modeled. These theoretical analyses have been performed for many years, but are only sporadically found in industry applications. The situation is different with fuzzy data analysis, whose use cases for game-related applications we will consider in Section 3.

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In Section 4 we review applications of fuzzy control in game AI which have been widely applied and proven to be useful in the implementation of more natural and believable game AI. Based on the state of the art in these three areas, we summarize the impact of fuzzy set theory on games and propose ideas for future research and applications in Section 5.

## 2. FUZZY GAME THEORY AND FUZZY GAMES

Game theory describes the study of mathematical models for the decision-making of multiple rational agents and their interactions in between them. Since its foundation [79] it has been widely studied and applied in areas of social sciences, economics, computer science, and many more. Game theory has developed into two main directions of research, non-cooperative and cooperative game theory. While the players in cooperative game theory can conclude binding contracts, the behaviors (including possible cooperation between players) in non-cooperative game theory are self-enforcing, i.e., they result from the players' self-interest without the possibility of binding contracts.

Traditional game theory is anchored on binary logic and the assumption that all players follow a rational behavior. Nevertheless, analysis with human players suggests, that they often do not act rationally and have limited access to or are different in the way they process information. As a result, fuzzy methods have been used to appropriately model these limitations. In the following, we will review concepts of fuzzy games [16, 23] and fuzzy game theory [17]. We will first analyze the developments in non-cooperative fuzzy games (Section 2.1) after which we will take a closer look at cooperative fuzzy games (Section 2.2).

**2.1. Non-cooperative Fuzzy Games.** The study of non-cooperative fuzzy games follows a long tradition. At its core, fuzzy extensions handle the imperfection of human judgment and imprecision in evaluation. These two principles form the main lines of research. While the first set of approaches uses fuzzy methods to model the preferences and beliefs of players, the second set is based on a fuzzy evaluation of the payoffs. In this section, we will introduce the underlying concepts of these fuzzy extensions. For readers, who are interested in the mathematical details of these approaches, we recommend the excellent survey by Larbani [64].

The approach introduced by Butnariu in 1978 [23] focuses on modeling the players' beliefs and preferences as fuzzy sets. In contrast to the classical definition by Neumann and Morgenstern [79] it removes the notion of a payoff function and replaces it with a general exchange of information, whereas information can be understood as any object or concept that can be transmitted in the interaction among the players. Billot later reformulated Butnariu's model [16] by introducing fuzzy lexicographic preferences. Those can be used to model a player's indecision between two very close options. The work by Buckley [22] introduced multiple goals to the decision-making process of two players. Since the players do not know the utility function of the other players, the agent's uncertainty of the other players' utility function is modeled by fuzzy sets. Players can use knowledge of past interactions to shape their belief of the opponent's mixed strategy.

A different fuzzification technique has been applied by Garagic and Cruz [50]. Here, the strategies are first fuzzified, which leads to the definition of a fuzzy preference matrix. Further defuzzification allows the determination of a Nash equilibrium [78] which is further considered to be the solution of the initial game. A fuzzification of payoffs and strategies has been proposed by Aristidou and Sarangi [9]. Due to its uniqueness, we also list the approach by Arfi [8] which makes use of linguistic fuzzy logic to complete a whole theory based on reasoning under fuzzy conditions. Arfi's definition includes linguistic fuzzy strategies, linguistic fuzzy preferences, and reasoning based on linguistic fuzzy logic. Furthermore, the interested reader is referred to the

book by Verma and Kumar [108], who present a collection of modern concepts for various types of non-cooperative fuzzy games.

In a different line of research fuzzy set theory has been used to model the fuzziness in the player's payoffs. This principle has first been extensively studied by Campos [27]. In his work, he expressed the fuzziness of the payoff matrix by replacing the crisp values with triangular fuzzy numbers. The resulting problem can be solved using linear programming. Many variances to solve the fuzzy matrix problem exist and have been extensively studied by Liu and Kao, Bector et al, Buckley and Jowers [21], Xu et al. [118] and many more. An overview of their mathematical differences and the linear programming techniques used can be found in the book by Bector and Chandra [12] and the aforementioned survey by Larbani [64].

**2.2. Cooperative Fuzzy Games.** While, in a binary cooperative game, a formed bond is certain, the players of fuzzy cooperative games can form fuzzy coalitions. The notion of fuzzy coalition has first been introduced by Aubin [10] in which players are allowed to join a coalition to some extent. The Shapley value [11, 99] indicates what payoff players can expect or should receive as a function of a coalition function. An explicit form of the Shapley function on a limited class of fuzzy games has been defined by Butnariu [24]. Tsurumi et al. argued that "most games of this class are neither monotone nondecreasing nor continuous with regard to rates of players' participation although crisp games are often considered to be monotone nondecreasing" [107]. To overcome this issue, they introduced a new class of fuzzy games with Choquet integral form in which any fuzzy game in the new class can be also derived from a crisp game. Similarly, the fuzzy games defined by Borkotokey [18] are both continuous and monotone nondecreasing with respect to the rate of the player's participation. They involve the fuzzification of coalitions and characteristic functions at the same time. Yu and Zhang [120] later proposed an extended form of the fuzzy games by Tsurumi et al. [107] which also involved the fuzzification of both components. A different approach has been taken by Mares [70] and Vlach [71]. Their game models involve crisp coalitions, but the coalition values of each player are fuzzy numbers.

Another interesting type of cooperative fuzzy games allows for coalitions in multiple attributes. Faigle [46] defined a game in which coalition among players are restricted to groups of players that share common attributes. The same asymmetry of agents has been studied by Fernández et al. [47] who proposed a Shapley kind of solution for soft games. Both types of games allow for the fuzziness of a coalition but consider their involvement in the players attributes to be crisp. Borkotokey et al. [19] extended this principle to player's who are able to participate partially in multiple coalitions and multiple attributes. The interested reader is referred to the survey by Borkotokey and Mesiar [20] who explicitly summarized studies on cooperative games under fuzzy environments.

### 3. FUZZY DATA ANALYSIS FOR GAMES

After this brief overview of fuzzy modeling in game theory, we would like to take a closer look at applications of fuzzy data analysis [58, 61] in real games. Here fuzzy data analysis is understood as the analysis of precise data with fuzzy methods [31].

One typical usage of data analysis in games can be found in match outcome prediction. Therefore, machine learning models are used to predict the winner before the actual end of the game or sometimes even before the match started. Fuzzy clustering and classification [14, 97] has been applied to multiple sports in that way, including football [96, 106], basketball [105], and baseball [65]. Fuzzy clustering, e.g. fuzzy c-means [15] and the various other extensions [58], allows points to be sorted into multiple clusters at once given their membership degree. As

a result, the grouping can be more natural than crisp clustering which forces elements to be part of a single group. Due to its simplicity and widespread use in the literature, fuzzy *c*-means seems to be the most commonly applied approach. Fuzzy classification [7] allows instances to be classified as being an element of multiple classes with varying degrees of membership. Both fuzzy clustering and classification, allow for a gradual result that can be used to highlight the model’s uncertainty in handling an instance in question.

In the works by Tsakonas et al. [106] and Rotsthein et al. [96] a fuzzy knowledge base has been constructed to predict winners of upcoming matches in the Ukrainian and Finnish football leagues. The models have further been tuned using genetic optimization and neural networks. According to the analysis by Rotsthein et al., both methods have resulted in “acceptable” simulation results, which had better prediction accuracy in case of uneven match-ups while being worse in identifying possible draws. About the same accuracy has been achieved by Tsakonas et al., who contrary to Rotsthein et al. observed that the genetic optimization performed slightly better. The work by Trawinski [105] tested fuzzy rule-based systems and compared their accuracy to linear regression prediction models. Out of the 10 evaluated models, only 3 performed better or equally good as linear regression in predicted instances of the test set. On the other hand, tested fuzzy rule-based systems have shown a tendency to produce simple rule sets using a low number of linguistic variables. The reduced complexity of the final model and the granularity provided by the use of linguistic variables results in a model that is easier to interpret than more complex regression models of the same accuracy. Another approach has been followed by Lee et al. [65], who made use of fuzzy time series to predict the win-rates of Major League Baseball teams.

Next to the prediction of future winners, we can use fuzzy models to analyze the behavior of players during a game. In a study by Stensrud and Gonzalez [101] fuzzy ARTMAP neural networks [28] have been used to analyze human game-play in multiple scenarios. The system aims to identify the low-level behaviors and map them to the stimuli within the human’s observation. Furthermore, it can be used to model learned high-level behaviors by creating a connection between the context and a set of executed low-level behaviors.

In addition to identifying or describing the players’ behavior, fuzzy models can also be used to measure their skill. An example is the skill classification system by Ćirović and Ćirović [30] who created game-play records to group players of the game Starcraft 2 into separate skill classes. Using 18 different features, the use of fuzzy *k*-NN classifier [59] has shown significantly better results in comparison to its crisp version. This may be due to the fuzziness of the concept “skill”. We will further address this topic in Section 4.3 in relation to adapting the difficulty of a game according to the player’s performance.

Analyzing the game-play cannot just be used to rate the human players’ game-play but also for improving the performance of an AI agent. The work by Aha et al. [6] demonstrated how case-based reasoning can be used to counter the strategy of other players effectively. A similar work by Cadena and Garrido [25] employed fuzzy case-based reasoning to select strategies in Starcraft. The fuzzy set approach has allowed the reasoner to abstract state information by grouping continuous feature values. This results in a simplified state space that eases the model learning procedure but could potentially result in the loss of important information. Due to the multiplicity of interesting AI approaches, we will spend the next section to cover many more agent development applications. As part of that, we will come back to the benefits of fuzzy methods on state encoding and belief state generation in Section 4.4.

#### 4. FUZZY LEARNING AND OPTIMIZATION OF AI AGENTS

By far the most common application of fuzzy set theory in the games industry has been the development of AI agents. Research in the area of game AI focuses on different aspects of AI agents, such as their performance, the believability of their behavior, and the challenge they pose to a human player. In the following, we will address these topics one after another in Section 4.1-Section 4.3. We additionally address the interesting application of modeling an agent's belief state in Section 4.4.

**4.1. Fuzzy Modeling of High-Performance AI Agents.** Fuzzy Modeling has led to many interesting applications in the field of AI development for games. Arguably, this is due to its many similarities with the field of fuzzy control [75, 124] that has apparent similarities with developing AI agents. Most notably, the Open Racing Car Simulator (TORCS) [117] brings together the fields of computer games research and classical control. The simulator gives information about the vehicle's position, its speed and distance to the current track segment, the track's curvature, and the relative position and velocity of nearby cars. Agents can control the car's speed and steer. Over the years, many car controllers have employed fuzzy control systems [45, 48, 49, 55, 82, 89, 98]. In most cases, the implemented agents relied on standard Takagi-Sugeno or Mamdani-Assilian controllers [102]. Despite controllers for speed and steering, some developers have implemented specialized controllers for e.g. blocking and overtaking opponents [83].

While early works have used hand-crafted controllers, later generations were optimized using evolutionary optimization schemes [5, 82, 89]. Next to reducing the agent's lap time per track, optimization goals include imitation learning, and the evolution of varying driving behaviors.

Similar techniques have been applied in other car racing games and simulators. The work by Etlik et al. [45] can be considered a bridge between simulated and real-world environments. In contrast, to the previous works, they have used image processing to identify the curvature of the upcoming track segment. It will be interesting to see, how much these methods will impact the development of autonomous cars and the perception of their environment.

Next to car racing games, other genres have seen a varying number of fuzzy AI solutions. Especially, the manifold real-time strategy (RTS) game genre has received many contributions in the form of specialized controllers. High-level decision making has been led to introducing fuzzy controllers in the game Starcraft. Given the collected information about the opponent's strategy, a fuzzy controller reactively selects a strategy that is executed by low-level controllers [92]. The fuzzy extension of the strategy selection component has notably increased the agent's performance. Nevertheless, the comparatively simple agent model has not been able to beat more advanced agents. In a similar work by Volna [110], the fuzzy extension has been shown to yield more stable results under consideration of uncertainties in the agent's partial observation of the game-state.

Another high-level strategy module empowered by fuzzy set theory is the team recommendation algorithm by Wang et al. [66, 113]. Here, a normalized order-based fuzzy integral has been learned to evaluate the strength of the opponent's army and the player's army effectiveness against it. Once an army has been trained, a combination of a potential field and fuzzy integrals has shown effective in solving the low-level task of unit movement [80]. Since the optimization procedure for learned fuzzy integrals have shown to be quite slow, fuzzy extreme learning machines have been applied to speed up the process [67]. The approach has been shown to speed up the model's training time and improve its accuracy in comparison to the classical approach using a genetic algorithm or particle swarm optimization.

The third big genre is first-person shooters. Here, agents are often trained to satisfy multiple goals such as staying behind cover, collecting ammunition and new weapons, as well as staying

alive and killing opponent players. Next to classical multi-objective optimization approaches, fuzzy control has shown to be an efficient candidate for agent development. According to a work by Waveren [115] the Quake bot Omicron, the Quake 2 bot Gladiator as well as his own Quake 3 Arena bot have used fuzzy logic to represent bot behaviors and their personalities. Those range from weapon usage preferences to item collection strategies.

Instead of hard-coding the agent’s behaviors, other studies have demonstrated the use of fuzzy finite state machines [94] to describe the agent’s basic behavior [43, 44] and improved it over time using evolutionary optimization. In contrast to crisp finite state machines, a state is represented as a collection of fuzzy variables. The fuzzy transition map defines the transitions from one state to another given the agent’s actions. Additionally, it has been demonstrated how an agent can adapt to its opponent in a relatively short amount of time using the Falcon architecture [112]. A similar process using fuzzy ART multi-channel Adaptive Resonance Map (ARAM) and Adaptive Resonance Theory Map (ARTMAP) [112] has been applied to the digital collectible card game Hearthstone [33]. Given enough training time, the agent was able to consistently beat a state-of-the-art Monte Carlo tree search agent. The approach has shown to generalize to previously unobserved game-states even in cases in which the opponent has played an unknown deck.

A technique we have not addressed yet is fuzzy reinforcement learning [52]. The generality of reinforcement learning has led to many successful applications in game AI. Despite, the existence of fuzzy extensions for reinforcement learning, they have not received much attention yet. Just a few works are mentioning fuzzy reinforcement learning in games, e.g. Ms. Pac-Man [35] and soccer [76]. It would be interesting to see how fuzzy extensions may impact modern deep reinforcement learning techniques. Recent studies on fuzzy reward shaping [29] and fuzzy reinforcement learning for continuous control [53] have shown faster learning times than crisp approaches and higher robustness to uncertainties in the environment. Only time will show if these early results transfer well to other applications.

**4.2. Creating Believable Agents.** Non-player characters (NPC) can be an important part of the gaming experience. With their behavior, they can shape the game world as well as become essential in the player’s interaction with it. Therefore, a common goal in the industry is the development of so-called believable agents, which are able to e.g. communicate, express emotions, and act naturally. Or in short, the intent is to make the NPCs behavior align with the expectations of the player [114].

In terms of communication, fuzzy set theory allows us to extend natural language processing in a way that binary logic would not allow [123]. Such systems have been applied for NPCs as receiver and sender of messages. As a receiver, the perception of emotions in text or facial expressions can be crucial in reacting appropriately to a given situation. For this purpose, fuzzy set theory has been applied to enhance a partner system for seven-stud poker with non-verbal communication [81, 88]. The system has been applied to choose facial expressions to react to the human player’s moves and to evaluate the current game-state using a fuzzy rule-based approach [87].

Another interesting application has been developed in the context of the game Party Quirks [69]. In this digital improvisational theater game, fuzzy concepts have been used to represent character prototypes. According to the works of Lakoff [62] as well as Rosch [95], such a prototype is defined as a collection of properties with varying degrees of membership. By applying fuzzy set theory, agents are able to model such concepts and yield more human-like characters.

Fuzzy methods have also been applied in sentiment analysis of game reviews to detect the overall sentiment [116]. The system outperformed non-fuzzy approaches in a sentiment analysis task including 2961 real mobile game review texts.

In terms of natural behavior, fuzzy finite state machines have been used successfully in the development of believable AI agents. In the 2k bot prize competition [56], a Turing test for believable first-person shooter agents in Unreal Tournament 2004, judges were asked to differentiate between game-play scenes of human players and AI agents. In 2008, the third rank has been achieved by an agent that used a reactive FALCON (fusion architecture for learning, cognition, and navigation) network [104]. The agent [111] learns knowledge in the form of cognitive nodes, which can be translated into a rule associating a pair of state and action to an estimated reward value. Remarkable is that the agent had the highest performance while being able to convince the most judges. Nevertheless, the judges' confidence rating has been slightly less than the rating for the first and second place.

A more general application has been presented by Acampora et al. [4]. In their work, they have proposed to use Timed Automata-based Fuzzy Controllers [1, 3] that unlike conventional fuzzy systems, model inference engine whose performance is strongly depending upon temporal concepts. By implementing theories for modeling emotions and personality, such as OCC [86] and OCEAN [73], they have developed an agent architecture that allows modeling an agent's internal state. This way, the agent's *emotions* and *personality traits* can be used to enhance video game bots from a human likeness point of view. Interestingly, the system has been developed using A Fuzzy Markup Language [2] which allows to define fuzzy systems in a hardware-independent way.

**4.3. Dynamic/Adaptive Difficulty.** As we mentioned before, performance does not make an AI agent fun to play against. It should provide the player with a decent challenge while keeping the game fair to both sides. The goal of dynamic difficulty adjustment, also known as adaptive difficulty adjustment, is to balance the game according to the player's skill, and therefore, create a fun and fair gaming experience which keeps the player engaged. Multiple researchers and developers have used fuzzy logic to measure the player's skill and adjust the AI agent's skill level [34, 54, 103] or the game's content [68]. More complex approaches made use of Evolutionary fuzzy cognitive maps (EFCM) [60, 72] to balance a runner-game in real-time based on the player's skill [36, 90]. According to Cai et al. [26] "EFCM models do not only model the fuzzy causal relationships among the variables, but also the probabilistic property of causal relationships, and asynchronous activity update of the concepts. With this model, the context variables evolve in a dynamic and uncertain manner." As a result, the game world of their interactive storytelling game exhibits more interesting dynamics.

Rather than adjusting the skill level of AI agents, this may also include adding new game elements or changing the game to suit the player's preferences. Interesting examples are games focusing on physical exercises (called Exergames). Here, the goal is to maximize the training's impact while keeping the player motivated. The work by Sinclair et al. [100] shows how fuzzy control can be used to alter the challenge of the game. Measuring the players' success in various subtasks as well as their heart rate allows them to not just alter the resistance of the bicycle pedals, but also the game's difficulty. The latter is achieved, by changing the gravity applied to the helicopter which is controlled by the player's movement. The system has shown able to adapt well to different player conditions and is effective in tuning the game such that the player's heart-rate matches the target heart-rate for optimal training.

Another approach has been used in the context of serious games. In a work, by Ponce et al. the goal was to keep players engaged by first identifying their personality traits and then

adapting the individual user-experience to maximize engagement [91]. A similar system has been used to vary the interface of an exergame for elderly people [74]. While personality traits are believed to be consistent over time, the emotional state of the user may also impact the user experience. Since those are also based on fuzzy concepts, fuzzy control systems [75] can and have been used to identify them. One example is a fuzzy control system applied to an educational video game [63], which has been shown to stimulate positive emotions and increased learning effectiveness.

**4.4. Fuzzy Modeling of Belief States.** State-determinization is a pressing topic in game AI. In the absence of crucial information, the observable variables are open for some interpretation by the agent. This phenomenon is typical for card games, in which the opponent’s cards cannot be seen, and strategy games, in which the so-called fog of war hides parts of the map.

In the context of the card game *Hearthstone* [39], fuzzy membership degrees have been used to determine the opponent’s deck of cards given the information of previously played cards and a database of common combinations [40, 41]. The approach has later been extended to predict the opponent’s next moves and play accordingly [38]. Evaluation of the prediction accuracy has shown that this approach is often able to correctly identify the opponent’s deck and, in later stages of the game, predict its moves correctly [37].

A similar approach has been used in the context of strategy games. After an opponent unit leaves the player’s observation and hides behind the fog of war, it is still on the map. Since it cannot be tracked, its position needs to be estimated for planning the movement of the player’s army. For this purpose, Yang et al. have used a fuzzy set theory-based single belief state generation method which incorporates the information of previous unit movements and the latest information on a unit’s whereabouts to predict its current position [119]. An evaluation in the  $\mu$ RTS environment [84], has shown that their approach increased the agent’s win-rate while being limited to scenarios that have been sufficiently represented in the training data.

## 5. CONCLUSIONS AND FUTURE DIRECTIONS

Fuzzy set theory has found its way in most areas of game theory and game development. Next to the many interesting models that are used in a research context, the application-oriented approaches have shown interesting results in the research environment. Especially simple models of fuzzy control and fuzzy clustering have shown many applications throughout game analysis and development. We suspect that the foundations of this widespread use lie in the simplicity of the methods, their robustness, and last but not least in their wide availability in software libraries. While we believe in the potential use-cases of fuzzy game theory, we are missing evidence for recent industrial applications in game development. A reason might be that the game-theoretic approaches are only needed for more intelligent agents and are therefore presumably not yet found in the games industry. Furthermore, a direct connection of the research in fuzzy game theory to industry-oriented applications is missing. Last but not least, benchmark scenarios are needed to demonstrate the performance of the methods and to arouse the interest of game manufacturers.

We want to end this survey by highlighting two applications in which fuzzy modeling may be the key technology to find a satisfying solution. In Section 4.2 we highlighted the many applications of fuzzy modeling for believable game AI. Despite the promising results in generating believable agents in a research context, we are not aware of modern industrial applications of fuzzy modeling of believable game AI. While deep reinforcement learning techniques have conquered even highly complex strategy games such as *Dota 2* [85] and *Starcraft 2* [109], the



resulting agents are optimized for performance. As Nareyek put it [77] the “outcome [of game AI] should be as believable and fun as possible”. Weaker but believable fuzzy AI agents may prove useful in reaching this goal. Research has shown that many of the tools that have been developed over the years, can be used to create realistic AI agents that excel in their believability. These agents are often less complex than deep learning models and should be adaptable to applications of the entertainment industry.

Secondly, we see interesting applications in game feel and game balancing research. Many game developers use the concept of *flow* to describe a “holistic sensation that people feel when they act with total involvement” [32]. The model was later revised by Ellis et al. [42] into a four-channel-model, including the two dimensions “skill” and “challenge”. Game developers and researchers alike had a hard time finding an objective measurement of flow. The apparent fuzziness of the concept may require the involvement of fuzzy measures to find a suitable description. Similar to successful applications like sentiment analysis and the detection of emotional states, fuzzy flow analysis may be an interesting research topic for future work.

We believe that fuzzy modeling will remain to be a key technology in the development of human-like and human-compatible AI. The need for explainable AI is a pressing topic in various disciplines [51, 57, 93] and research on fuzzy methods in games already shows how fuzzy models can improve reasoning under fuzzy conditions while allowing for interpretable solutions. We hope that research on games will keep its connection to other areas and remain a driver for innovation in AI technologies as it has been in the past.

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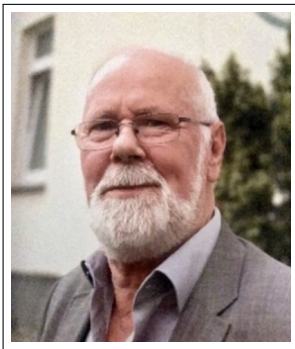
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